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Kalman filter-based adaptive control for networked systems with unknown parameters and randomly missing outputs

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SUMMARY

This paper investigates the problem of adaptive control for networked control systems with unknown model parameters and randomly missing outputs. In particular, for a system with the autoregressive model with exogenous input placed in a network environment, the randomly missing output feature is modeled as a Bernoulli process. Then, an output estimator is designed to online estimate the missing output measurements, and further a Kalman filter-based method is proposed for parameter estimation. Based on the estimated output and the available output, and the estimated model parameters, an adaptive control is designed to make the output track the desired signal. Convergence properties of the proposed algorithms are analyzed in detail. Simulation examples illustrate the effectiveness of the proposed method. Copyright © 2008 John Wiley & Sons, Ltd.

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KEY WORDS: networked control systems (NCSs); limited feedback information; randomly missing outputs; adaptive control; Kalman filter

1. INTRODUCTION

Networked control systems (NCSs) are a type of distributed control systems, where the information of control system components (reference input, plant output, control input, etc.) is exchanged via communication networks. Owing to the introduction of networks, NCSs have many attractive advantages, such as reduced system wiring, low weight and space, ease of system diagnosis and maintenance, and increased system agility, which motivated the research in NCSs. The study of

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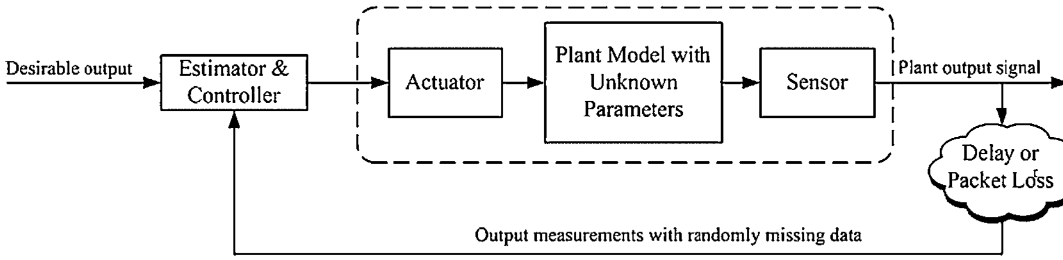

Figure 1. An NCS with randomly missing outputs.

NCSs has been an active research area in the past several years, see some recent survey articles [1–3] and the references therein. On the other hand, the introduction of networks also presents some challenges such as the *limited feedback information* caused by packet transmission delays and packet loss; both of them are due to the sharing and competition of the transmission medium, and bring difficulties for analysis and design for NCSs. The information transmission delay arises from by the limited capacity of the communication network used in a control system, whereas the packet loss is caused by the unavoidable data losses or transmission errors. Both the information transmission delay and packet loss may result in randomly missing output measurements at the controller node, as shown in Figure 1. So far different approaches have been used to characterize the limited feedback information. For example, the information transmission delay and packet losses have been modeled as Markov chains [4]. The binary Bernoulli distribution is used to model the packet losses in [5–7].

The main challenge of NCS design is the *limited feedback information* (information transmission delays and packet losses), which can degrade the performance of systems or even cause instability. Various methodologies have been proposed for modeling, stability analysis, and controller design for NCSs in the presence of limited feedback information. A novel feedback stabilization solution of multiple coupled control systems with limited communication is proposed by bringing together communication and control theoretical issues in [8]. Further, the control and communication co-design methodology are applied in [9, 10]—a method of stabilizing linear NCSs with medium access constraints and transmission delays by designing a delay-compensated feedback controller and an accompanying medium access policy is presented. In [11], the relationship of sampling time and maximum allowable transfer interval to keep the systems stable is analyzed by using a stability region plot; the stability analysis of NCSs is addressed by using a hybrid system stability analysis technique. In [12], a new NCS protocol, try-once-discard, which employs dynamic scheduling method, is proposed and the analytic proof of global exponential stability is provided based on Lyapunov's second method. In [13], the conditions under which NCSs subject to dropped packets are mean square stable are provided. Output feedback controller that can stabilize the plant in the presence of delay, sampling, and dropout effects in the measurement and actuation channels is developed in [14]. In [15], the authors model the NCSs with packet dropout and delays as ordinary linear systems with input delays and further design state feedback controllers using Lyapunov–Razumikhin function method for the continuous-time case, and Lyapunov–Krasovskii-based method for the discrete-time case, respectively. In [16], the time delays and packet dropout are simultaneously considered for state feedback controller design based on a delay-dependent approach; the maximum allowable value of the network-induced delays can be determined by

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solving a set of linear matrix inequalities. Most recently, Gao and Chen, for the first time, incorporate simultaneously three types of communication limitation, e.g. measurement quantization, signal transmission delay, and data packet dropout into the NCS design for robust H_∞ state estimation [17], and passivity-based controller design [18], respectively. Further, a new delay system approach that consists of multiple successive delay components in the state, is proposed and applied to network-based control in [19].

However, the results obtained for NCSs are still limited: most of the aforementioned results assume that the plant is given and model parameters are available, while few papers address the analysis and synthesis problems for NCSs whose plant parameters are *unknown*. In fact, while controlling a real plant, the designer rarely knows its parameters accurately [20]. To the best of our knowledge, adaptive control for systems with unknown parameters and randomly missing outputs in a network environment has not been fully investigated, which is the focus of this paper.

It is worth noting that systems with *regular* missing outputs—a special case of those with randomly missing outputs—can also be viewed as multirate systems, which have uniform but various input/output sampling rates [21]. Such systems may have regular-output-missing feature. In [22], Ding and Chen use an auxiliary model and a modified recursive least-squares algorithm to realize simultaneous parameter and output estimation of dual-rate systems. Further, a least-squares-based self-tuning control scheme is studied for dual-rate linear systems [23] and nonlinear systems [24], respectively. However, network-induced limited feedback information unavoidably results in *randomly* missing output measurements. To generalize and extend the adaptive control approach for multirate systems [23, 24] to NCSs with randomly missing output measurements and unknown model parameters is another motivation of this work.

In this paper, we first model the availability of output as a Bernoulli process. Then, we design an output estimator to online estimate the missing output measurements, and further propose a novel Kalman filter-based method for parameter estimation with randomly output missing. Based on the estimated output or the available output, and the estimated model parameters, an adaptive control is proposed to make the output track the desired signal. Convergence of the proposed output estimation and adaptive control algorithms is analyzed.

The rest of this paper is organized as follows. The problem of adaptive control for NCSs with unknown model parameters and randomly missing outputs is formulated in Section 2. In Section 3, the proposed algorithms for output estimation, model parameter estimation, and adaptive control are presented. In Section 4, the convergence properties of the proposed algorithms are analyzed. Section 5 gives several illustrative examples to demonstrate the effectiveness of the proposed algorithms. Finally, concluding remarks are given in Section 6.

Notations: The notations used throughout the paper are fairly standard. 'E' denotes the expectation. The superscript 'T' stands for matrix transposition, $\lambda_{\max/\min}(X)$ represents the maximum/minimum eigenvalue of X, $|X| = \det(X)$ is the determinant of a square matrix X, $\|X\|^2 = \mathrm{tr}(XX^{\mathrm{T}})$ stands for the trace of XX^{T}. If $\exists\,\delta_0 \in \mathrm{R}^+$ and $k_0 \in \mathrm{Z}^+$, $|f(k)| \leqslant \delta_0 g(k)$ for $k \geqslant k_0$, then $f(k) = O(g(k))$, if $f(k)/g(k) \to 0$ for $k \to \infty$, then $f(k) = o(g(k))$.

2. PROBLEM FORMULATION

The problem of interest in this work is to design an adaptive control scheme for networked systems with unknown model parameters and randomly missing outputs. In Figure 2, the output

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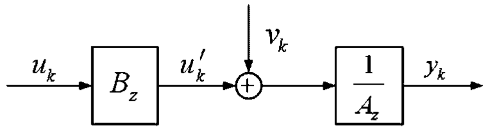

Figure 2. ARX model structure.

measurements y_k could be unavailable at the controller node at some time instants because of the network-induced limited feedback information, e.g. transmission delay and/or packet loss. The data transmission protocols like TCP guarantee the delivery of data packets in this way: when one or more packets are lost the transmitter retransmits the lost packets. However, since a retransmitted packet usually has a long delay that is not desirable for control systems, the retransmitted packets are outdated by the time they arrive at the controller [13, 25]. Therefore, in this paper, it is assumed that the output measurements that are delayed in transmission are regarded as missed ones.

The availability of y_k can be viewed as a random variable γ_k. γ_k is assumed to have Bernoulli distribution:

$$E(\gamma_k \gamma_s) = E\gamma_k E\gamma_s \quad \text{for } k \neq s$$
$$\text{Prob}(\gamma_k) = \begin{cases} \mu_k & \text{if } \gamma_k = 1 \\ 1-\mu_k & \text{else if } \gamma_k = 0 \end{cases} \tag{1}$$

where $0 < \mu_k \leqslant 1$.

Consider a single-input–single-output (SISO) process (Figure 2):

$$A_z y_k = B_z u_k + v_k \tag{2}$$

where u_k is the system input, y_k the output, and v_k the disturbing white noise with variance r_v. A_z and B_z are two backshift polynomials defined as

$$A_z = 1 + a_1 z^{-1} + a_2^{-2} + \cdots + a_{n_a} z^{-n_a}$$
$$B_z = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}$$

The polynomial orders n_a and n_b are assumed to be given. Equation (2) can be written equivalently as the following linear regression model:

$$y_k = \varphi_{0k}^{\mathrm{T}} \theta + v_k \tag{3}$$

where

$$\varphi_{0k} = [-y_{k-1} \ \ -y_{k-2} \ \ \cdots \ \ -y_{k-n_a} \ \ u_k \ \ u_{k-1} \ \ \cdots \ \ u_{k-n_b}]^{\mathrm{T}}$$
$$\theta = [a_1 \ \ a_2 \ \ \cdots \ \ a_{n_a} \ \ b_0 \ \ b_1 \ \ \cdots \ \ b_{n_b}]^{\mathrm{T}}$$

Vector φ_{0k} represents system's excitation and response information necessary for parameter estimation, whereas vector θ contains model parameters to be estimated.

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For a system with the autoregressive model with exogenous input (ARX) placed in a networked environment subject to randomly missing outputs, the objectives of this paper are:

1. Design an output estimator to online estimate the missing output measurements.
2. Develop a recursive Kalman filter-based identification algorithm to estimate the unknown model parameters.
3. Propose an adaptive tracking controller to make the system output track a given desired signal.
4. Analyze the convergence properties of the proposed algorithms.

3. PARAMETER ESTIMATION, OUTPUT ESTIMATION, AND ADAPTIVE CONTROL DESIGN

There are two main challenges of the adaptive control design for a networked system, as depicted in Figure 1: (1) randomly missing output measurements; (2) unknown system model parameters. Therefore, in this section, we first propose algorithms for missing output estimation and unknown model parameter estimation, and then design the adaptive control scheme.

3.1. Parameter estimation and missing output estimation

Consider the model in (3). It is shown by [26, 27] that the corresponding Kalman filter-based parameter estimation algorithm is given by

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_{0k}(y_k - \varphi_{0k}^{\mathrm{T}} \hat{\theta}_{k-1}) \tag{4}$$

$$K_{0k} = \frac{P_{0k-1}\varphi_{0k}}{r_v + \varphi_{0k}^{\mathrm{T}} P_{0k-1}\varphi_{0k}} \tag{5}$$

$$P_{0k} = P_{0k-1} - \frac{P_{0k-1}\varphi_{0k}\varphi_{0k}^{\mathrm{T}} P_{0k-1}}{r_v + \varphi_{0k}^{\mathrm{T}} P_{0k-1}\varphi_{0k}} \tag{6}$$

where $\hat{\theta}_k$ represents the estimated parameter vector at instant k, and P_{0k} is the covariance of $\hat{\theta}_k$.

It is worth to note that the above algorithm as shown in (4)–(6) cannot be *directly* applied to the parameter estimation of systems with randomly missing outputs in a network environment, as y_k in (4) may not be available. This motivates us to develop a new algorithm that can simultaneously online estimate the unavailable missing output and estimate system parameters under the network environment. The proposed algorithm consists of two steps.

Step 1: *Output estimation*. Albertos *et al.* propose a simple algorithm that uses the input–output model, replacing the unknown past values by estimates when necessary [28]. Inspired by this work, we design the following output estimator:

$$z_k = \gamma_k y_k + (1-\gamma_k)\hat{y}_k \tag{7}$$

where

$$\hat{y}_k = \varphi_k^{\mathrm{T}} \hat{\theta}_{k-1}$$

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and

$$\varphi_k = [-z_{k-1} \ \cdots \ -z_{k-n_a} \ u_k \ u_{k-1} \ \cdots \ u_{k-n_b}]^{\mathrm{T}}$$

In (7), γ_k is a Bernoulli random variable used to characterize the availability of y_k at time instant k at the controller node, as defined in (1). With the time-stamp technique, the controller node can detect the availability of the output measurements, and thus, the values of $\gamma_k s$ (either 1 or 0) are known. The knowledge of their corresponding probability $\mu_k s$ is not used in the designed estimator. The structure of the designed output estimator is intuitive and simple yet very effective, which will be seen soon from the simulation examples.

Step 2: *Model parameter estimation*. Replacing y_k and φ_{0k} in the algorithm (4)–(6) by z_k and φ_k, respectively, and considering the random variable γ_k, we readily obtain the following algorithm:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k(z_k - \varphi_k^{\mathrm{T}}\hat{\theta}_{k-1}) \tag{8}$$

$$K_k = \frac{P_{k-1}\varphi_k}{r_v + \varphi_k^{\mathrm{T}} P_{k-1}\varphi_k} \tag{9}$$

$$P_k = P_{k-1} - \gamma_k \frac{P_{k-1}\varphi_k\varphi_k^{\mathrm{T}} P_{k-1}}{r_v + \varphi_k^{\mathrm{T}} P_{k-1}\varphi_k} \tag{10}$$

Remark 3.1

Consider two extreme cases. If the availability sequence $\{\gamma_1, \ldots, \gamma_k\}$ constantly assumes 1, then no output measurement is lost, and the algorithm above will reduce to the algorithm (4)–(6). On the other hand, if the availability sequence $\{\gamma_k\}$ constantly takes zero, then all output measurements are lost, and the parameter estimates just keep the initial values.

3.2. *Adaptive control design*

Consider the tracking problem. Let $y_{r,k}$ be a desired output signal, and define the output tracking error as

$$\zeta_k := y_k - y_{r,k}$$

If the control law u_k is appropriately designed such that $y_{r,k} = \varphi_{0k}^{\mathrm{T}}\theta$, then the tracking error ζ_k approaches zero finally. Replacing θ by $\hat{\theta}_{k-1}$ and φ_{0k} by φ_k yields

$$\begin{aligned} y_{r,k} = \varphi_k^{\mathrm{T}}\hat{\theta}_{k-1} &= -\sum_{i=1}^{n_a} \hat{\theta}_{i,k-1} z_{k-i} + \sum_{i=0}^{n_b} \hat{\theta}_{n_a+i+1,k-1} u_{k-i} \\ &= -\hat{a}_{1,k-1} z_{k-1} - \cdots - \hat{a}_{n_a,k-1} z_{k-n_a} + \hat{b}_{0,k-1} u_k + \cdots + \hat{b}_{n_b,k-1} u_{k-n_b} \end{aligned}$$

Therefore, the control law can be designed as

$$u_k = \frac{1}{\hat{b}_{0,k-1}} \left[y_{r,k} + \sum_{i=1}^{n_a} \hat{a}_{i,k-1} z_{k-i} - \sum_{i=1}^{n_b} \hat{b}_{i,k-1} u_{k-i} \right] \tag{11}$$

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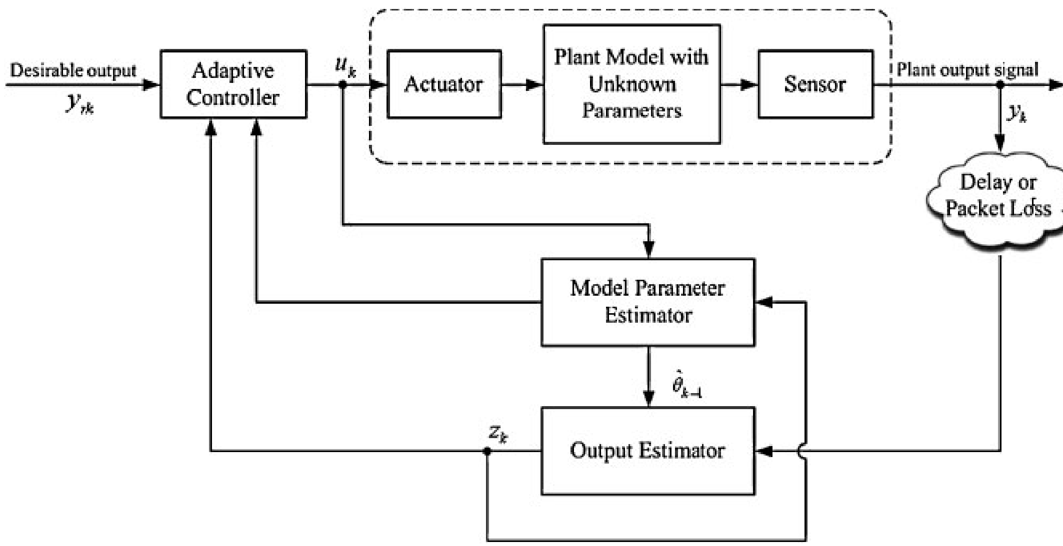

Figure 3. Adaptive control diagram.

The proposed adaptive control scheme consists of the missing output estimator [Equation (7)], model parameter estimator [Equations (8)–(10)], and the adaptive control law [Equation (11)]. The overall control diagram is shown in Figure 3.

4. CONVERGENCE ANALYSIS

This section focuses on the analysis of some convergence properties. Some preliminaries are first summarized to facilitate the following convergence analysis of parameter estimation in (8)–(10) and of output estimation in (7). Inspired by [22, 24, 29], the convergence analysis is carried out under the stochastic framework.

4.1. Preliminaries

To facilitate the convergence analysis, directly applying the matrix inversion formula [30]

$$(A+BCD)^{-1}=A^{-1}-A^{-1}B(C^{-1}+DA^{-1}B)^{-1}DA^{-1}$$

the proposed parameter estimation algorithm in Section 3.1 [(8)–(10)] can be equivalently rewritten as

$$\hat{\theta}_k=\hat{\theta}_{k-1}+r_v^{-1}P_k\varphi_k(z_k-\varphi_k^{\mathrm{T}}\hat{\theta}_{k-1}) \tag{12}$$

$$P_k^{-1}=P_{k-1}^{-1}+\gamma_k r_v^{-1}\varphi_k\varphi_k^{\mathrm{T}} \tag{13}$$

Suppose that P_k is initialized by $p_0 I$, where p_0 is a positive real value large enough, and define $r_k = \mathrm{tr}(P_k^{-1})$. The relation between r_k and $|P_k^{-1}|$ can be established in the following lemma:

Lemma 4.1
The following relation holds:

$$\ln E|P_k^{-1}| = O(\ln E r_k) \tag{14}$$

Proof
Using the formulae

$$\mathrm{tr}(X) = \sum_{i=1}^{n} \lambda_i(X) \quad \text{and} \quad |X| = \prod_{i=1}^{n} \lambda_i(X)$$

where n is the dimension of X, we have

$$E|P_k^{-1}| \leqslant (E r_k)^n$$

This completes the proof. □

The next lemma shows the convergence of two infinite series that will be useful later.

Lemma 4.2
The following inequalities hold:

$$\sum_{i=1}^{t} \mu_i r_v^{-1} E(\varphi_i^{\mathrm{T}} P_i \varphi_i) \leqslant \ln E|P_k^{-1}| + n_0 \ln p_0 \text{ a.s.} \tag{15}$$

$$\sum_{i=1}^{\infty} \mu_i r_v^{-1} \frac{E(\varphi_i^{\mathrm{T}} P_i \varphi_i)}{(\ln E|P_i^{-1}|)^c} < \infty \text{ a.s.} \tag{16}$$

where $c > 1$.

Proof
The proof can be done along the similar way as Lemma 2 in [23] and is omitted here. □

The following is the well-known martingale convergence theorem that lays the foundation for the convergence analysis of the proposed algorithms.

Theorem 4.1 (*Goodwin and Sin* [*31*])
Let $\{X_k\}$ be a sequence of nonnegative random variables adapted to an increasing σ-algebras $\{\mathscr{F}_k\}$. If

$$E(X_{k+1}|\mathscr{F}_k) \leqslant (1+\varepsilon_k) X_k - \alpha_k + \beta_k \text{ a.s.}$$

where $\alpha_k \geqslant 0$, $\beta_k \geqslant 0$, and $E X_0 < \infty$, $\sum_{i=0}^{\infty} |\varepsilon_i| < \infty$, $\sum_{i=0}^{\infty} \beta_i < \infty$ almost surely (a.s.), then X_k converges a.s. to a finite random variable and

$$\lim_{N \to \infty} \sum_{i=0}^{N} \alpha_i < \infty \text{ a.s.}$$

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4.2. Convergence analysis

To carry out the convergence analysis of the proposed algorithms, it is essential to appropriately construct a martingale process satisfying the conditions of Theorem 4.1. Main results on the convergence properties of the proposed algorithm are summarized in the following theorem:

Theorem 4.2
For the system considered in (3), assume that

(A1) $\{v_k, \mathscr{F}_k\}$ is a martingale difference sequence satisfying

$$E(v_k|\mathscr{F}_{k-1}) = 0 \text{ a.s.} \tag{17}$$

$$E(v_k^2|\mathscr{F}_{k-1}) = r_v < \infty \text{ a.s.} \tag{18}$$

(A2) B_z is stable, i.e. zeros of B_z are inside the closed unit disk.

Suppose the desired output signal is bounded: $|y_{r,k}| < \infty$. Applying the missing output estimator [Equation (7)], model parameter estimator [Equations (8)–(10)], and the adaptive control law [Equation (11)], then the output tracking error has the property of minimum variance, i.e.

(1) $$\lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} (y_{r,i} - y_i + v_i)^2 = 0 \text{ a.s.}$$

(2) $$\lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} \mu_i E\{(z_i - y_{r,i})^2 | \mathscr{F}_{i-1}\} = r_v < \infty \text{ a.s.}$$

Proof
As pointed out in [29] and [31], from (A2) it follows that:

$$\frac{1}{k} \sum_{i=1}^{k} u_i^2 \leqslant O(1) + O\left(\frac{c_1}{k} \sum_{i=1}^{k} y_i^2\right) \text{ a.s.} \tag{19}$$

Here, c_1 is a positive constant. Define the parameter estimation error vector and residual scalar as

$$\tilde{\theta}_k = \hat{\theta}_k - \theta$$

$$\eta_k = y_{r,k} - y_k + v_k$$

By (18) we have

$$\frac{1}{k} \sum_{i}^{k} y_i^2 = O(1) + O\left(\frac{1}{k} \sum_{i}^{k} \eta_i^2\right) \tag{20}$$

From (7) and (8), it can be easily verified that

$$\begin{aligned}\tilde{\theta}_k &= \tilde{\theta}_{k-1} + r_v^{-1} P_k \varphi_k (z_k - \hat{y}_k) \\ &= \tilde{\theta}_{k-1} + r_v^{-1} P_k \varphi_k \gamma_k (y_k - \hat{y}_k) \\ &= \tilde{\theta}_{k-1} + r_v^{-1} P_k \varphi_k \gamma_k (-\eta_k + v_k)\end{aligned} \tag{21}$$

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Define a Lyapunov function V_k as

$$V_k = \tilde{\theta}_k^{\mathrm{T}} P_k^{-1} \tilde{\theta}_k \tag{22}$$

Note that V_k is nonnegative as P_k^{-1} is a nonnegative definite matrix. From (13) and (21), (22) can be further evaluated as

$$\begin{aligned} V_k &= \tilde{\theta}_{k-1}^{\mathrm{T}} P_k^{-1} \tilde{\theta}_{k-1}^{-1} + 2r_v^{-1}\gamma_k(-\eta_k + v_k)\varphi_k^{\mathrm{T}}\tilde{\theta}_{k-1} + r_v^{-2}\gamma_k(-\eta_k+v_k)^2\varphi_k^{\mathrm{T}} P_k \varphi_k \\ &= V_{k-1} + r_v^{-1}\gamma_k(\varphi_k^{\mathrm{T}}\tilde{\theta}_{k-1})^2 + 2r_v^{-1}\gamma_k(-\eta_k+v_k)\varphi_k^{\mathrm{T}}\tilde{\theta}_{k-1} \\ &\quad + r_v^{-2}\gamma_k(-\eta_k+v_k)^2\varphi_k^{\mathrm{T}} P_k\varphi_k \\ &= V_{k-1} + r_v^{-1}\gamma_k\eta_k^2 + 2r_v^{-1}\gamma_k(-\eta_k+v_k)\eta_k + r_v^{-2}\gamma_k(-\eta_k+v_k)^2\varphi_k^{\mathrm{T}} P_k\varphi_k \\ &= V_{k-1} - r_v^{-1}\gamma_k(1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k\varphi_k)\eta_k^2 + 2r_v^{-1}\gamma_k(1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k\varphi_k)\eta_k v_k \\ &\quad + r_v^{-2}\gamma_k\varphi_k^{\mathrm{T}} P_k\varphi_k v_k^2 \end{aligned} \tag{23}$$

Taking expectation with respect to $\mathscr{F}_{k-1}$ on both sides of (23) gives

$$E(V_k|\mathscr{F}_{k-1}) \leqslant V_{k-1} - r_v^{-1}\mu_k(1 - r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k\varphi_k))\eta_k^2 + r_v^{-3}\mu_k E(\varphi_k^{\mathrm{T}} P_k\varphi_k) \tag{24}$$

Define a new sequence:

$$W_k = \frac{V_k}{(\ln E|P_k^{-1}|)^c}, \quad c>1 \tag{25}$$

Since $\ln E|P_k^{-1}|$ is nondecreasing, it follows from (24) and (25) that

$$\begin{aligned} E(W_k|\mathscr{F}_{k-1}) &\leqslant \frac{V_{k-1}}{(\ln E|P_k^{-1}|)^c} - \frac{r_v^{-1}\mu_k(1 - r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k\varphi_k))\eta_k^2}{(\ln E|P_k^{-1}|)^c} \\ &\quad + r_v^{-2}\frac{\mu_k r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k\varphi_k)}{(\ln E|P_k^{-1}|)^c} \\ &\leqslant W_{k-1} - \frac{r_v^{-1}\mu_k(1 - r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k\varphi_k))\eta_k^2}{(\ln E|P_k^{-1}|)^c} \\ &\quad + r_v^{-2}\frac{\mu_t r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k\varphi_k)}{(\ln E|P_k^{-1}|)^c} \end{aligned} \tag{26}$$

From (10) we have

$$1 - r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k\varphi_k) > 0$$

In addition, note that by Lemma 4.2 the summation of the third term in (26) from 0 to ∞ is finite. Therefore, Theorem 4.1 is applicable, and it yields

$$\sum_{k=1}^{\infty} \frac{r_v^{-1}\mu_k(1 - r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k\varphi_k))\eta_k^2}{(\ln E|P_k^{-1}|)^c} < \infty \text{ a.s.} \tag{27}$$

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Further, Lemma 4.1 indicates

$$\sum_{k=1}^{\infty} \frac{r_v^{-1}\mu_k(1-r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k \varphi_k))\eta_k^2}{(\ln Er_k)^c} < \infty \text{ a.s.} \tag{28}$$

As $[1-r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k \varphi_k)]$ is positive and nondecreasing, it holds that $1 = O[1-r_v^{-1}E(\varphi_k^{\mathrm{T}} P_k \varphi_k)]$. Hence,

$$\sum_{i=1}^{\infty} \frac{\eta_i^2}{(\ln Er_i)^c} < \infty \text{ a.s.} \tag{29}$$

Since $\lim_{k\to\infty} \ln Er_k = \infty$, then from the Kronecker lemma [31], it follows that:

$$\lim_{k\to\infty} \Delta_k = 0 \text{ a.s.}$$

where

$$\Delta_k \triangleq \frac{1}{(\ln Er_k)^c} \sum_{i=1}^{k} \eta_i^2$$

With

$$r_k = \frac{n}{p_0} + \sum_{i=1}^{k} r_v^{-1} \gamma_i \varphi_i^{\mathrm{T}} \varphi_i$$

and (19), we obtain

$$\begin{aligned}
\frac{1}{k}\sum_{i=1}^{k} \eta_i^2 &= \frac{\Delta_k}{k} O((\ln Er_k)^c) \\
&= \frac{\Delta_k}{k} O(r_k) \\
&= \frac{\Delta_k}{k} O\left(\frac{n}{p_0} + n_a \sum_{i=1}^{k} \gamma_i z_i^2 + n_b \sum_{i=1}^{k} \gamma_i u_i^2\right) \\
&= \Delta_k O\left(\frac{1}{k}\sum_{i=1}^{k} y_i^2\right)
\end{aligned} \tag{30}$$

Substituting (30) into (20) gives

$$\frac{1}{k}\sum_{i}^{k} y_i^2 = O(1) \text{ a.s.}$$

which implies together with (30) that

$$\lim_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} \eta_i^2 = 0 \text{ a.s.}$$

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or equivalently

$$\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}(y_{r,i}-y_i+v_i)^2=0 \text{ a.s.} \tag{31}$$

Since

$$\begin{aligned}E\{(y_{r,k}-y_k+v_k)^2|\mathscr{F}_{k-1}\} &= E[(y_{r,k}-y_k)^2+2y_{r,k}v_k-2y_kv_k+v_k^2|\mathscr{F}_{k-1}]\\ &= E[(y_{r,k}-y_k)^2|\mathscr{F}_{k-1}]+0-2r_v+r_v\\ &= E[(y_{r,k}-y_k)^2|\mathscr{F}_{k-1}]-r_v \text{ a.s.}\end{aligned}$$

and $\gamma_k z_k=\gamma_k y_k$, we have

$$\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\mu_i E\{(z_i-y_{r,i})^2|\mathscr{F}_{i-1}\}=\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\mu_i E\{(y_i-y_{r,i})^2|\mathscr{F}_{i-1}\}=r_v \text{ a.s.}$$

This completes the proof. □

5. ILLUSTRATIVE EXAMPLES

In this section, we give three examples to illustrate the adaptive control design scheme proposed in the previous sections.

The ARX model shown in Figure 2 in the simulation is chosen as

$$(1+a_1z^{-1}+a_2z^{-2})y_k=(b_0+b_1z^{-1}+b_2z^{-2})u_k+v_k$$

which is assumed to be placed in a network environment (Figure 1) with randomly missing output measurements and unknown model parameters. $\{v_k\}$ is a Gaussian white noise sequence with zero mean and variance $r_v=0.05^2$. The parameter vector $\theta=[a_1\ a_2\ b_0\ b_1\ b_2]^{\mathrm{T}}$ is to be estimated. Here, true values of θ are

$$\theta=[-1.5\ \ 0.7\ \ 0.5\ \ 0.2\ \ 0.34]^{\mathrm{T}}$$

For simulation purposes, we assume that: (1) θ is unknown and initialized by ones; (2) the output measurement $\{y_k\}$ is subject to *randomly* missing when transmitted to the controller node; (3) the availability of the output measurements (y_k) at the controller node is characterized by the probability μ_k; (4) the desired output signal to be tracked is a square wave alternating between -1 and 1 with a period of 1000. Mathematically, it is given by

$$y_{r,(500i+j)}=(-1)^{i+1},\quad i=0,1,2,\ldots,\quad j=1,2,\ldots,500$$

In the following simulation studies, we carry out experiments for three different scenarios regarding the availability of the output measurements at the controller node and the parameter variation, and examine the control performance, respectively. According to the proposed adaptive control scheme shown in Figure 3, we apply the algorithms of the missing output estimator, model parameter estimator, and the adaptive control law to the NCS.

Example 1
$\mu_k = 0.85$. In the first example, 85% of all the measurements are available at the controller node after network transmission from the sensor to the controller. The output response is shown in Figure 4, from which it is observed that the output tracking performance is satisfactory. In order to take a closer observation on the model parameter estimation and output estimation, we define the relative parameter estimation error as

$$\delta_{\text{par}}\% = \frac{\|\hat{\theta}_k - \theta\|}{\|\theta\|} \times 100\%$$

It is shown in Figure 5 (solid curve) that $\delta_{\text{par}}\%$ is becoming smaller with k increasing. Comparison between the estimated outputs and true outputs during the time range $501 \leqslant t \leqslant 550$ is illustrated in Figure 6: The dashed lines are corresponding to the time instants when data missing occurs, and the small circles on the top of the dashed lines represent the estimated outputs at these time instants. From Figure 6 it can be found that the missing output estimation also exhibits good performance.

Example 2
$\mu_k = 0.65$. In the second example, a worse case subject to more severe randomly missing outputs is examined: only 65% of all the measurements are available at the controller node. The output response is shown in Figure 7. Even though the available output measurements are more scarce than those in Example 1, it is still observed that the output is tracking the desired signal with satisfactory performance. The relative parameter estimation error, $\delta_{\text{par}}\%$, is shown in Figure 5 (dashed curve). Clearly, it is decreasing when k is increasing. The estimated outputs and the true outputs are illustrated in Figure 8, from which we can see good output estimation performance.

For the comparison purpose, the relative parameter estimation errors of these two examples are shown in Figure 5. We can see that the parameter estimation performance when $\mu_k = 0.85$ is better

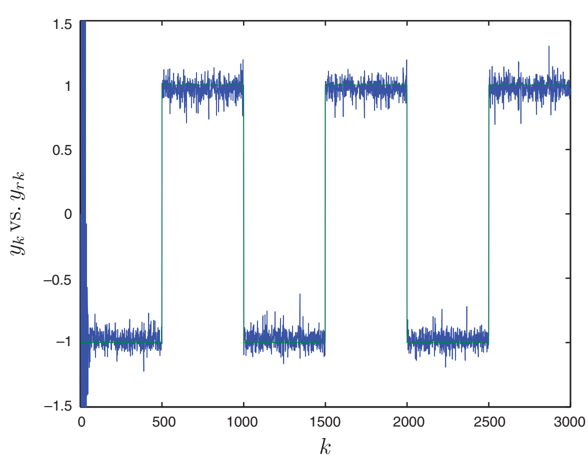

Figure 4. Example 1: Output response when $\mu_k = 0.85$.

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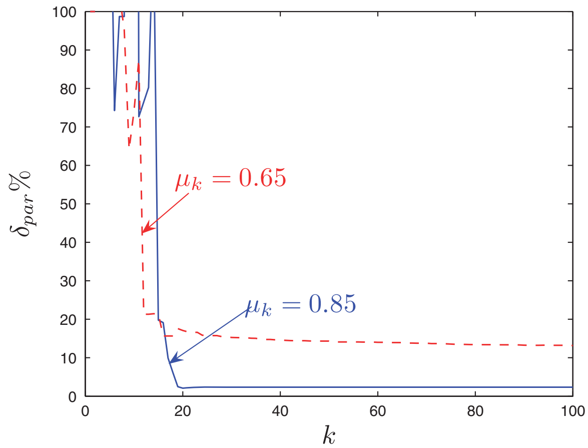

Figure 5. Comparison of relative parameter estimation errors for Example 1 and Example 2: solid line for Example 1; dotted line for Example 2.

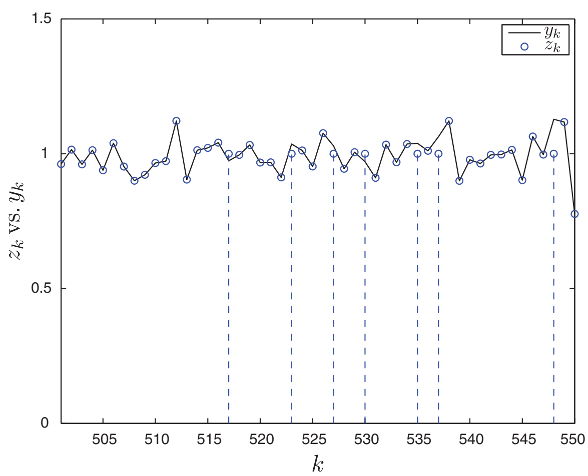

Figure 6. Example 1: Comparison between estimated and true outputs when $\mu_k = 0.85$. (The dashed line represents output missing.)

than that when $\mu_k = 0.65$. It is no doubt that the estimation performance largely depends on data completeness that is characterized by μ_k.

Example 3

Output tracking performance subject to parameter variation. In practice, the model parameters may vary during the course of operation due to the change of load, external disturbance, noise, and so on. Hence, it is also paramount to explore the robustness of the designed controller against the influence of parameter variation. In this example, we assume that at $k = 1500$, model parameters are all increased by 50%. The output response is shown in Figure 9. It can be seen

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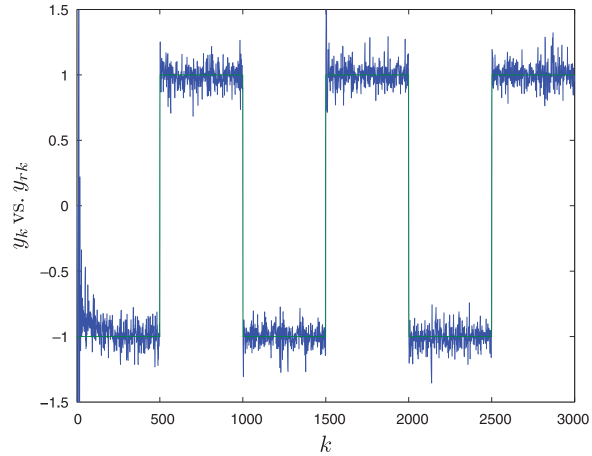

Figure 7. Example 2: Output response when $\mu_k = 0.65$.

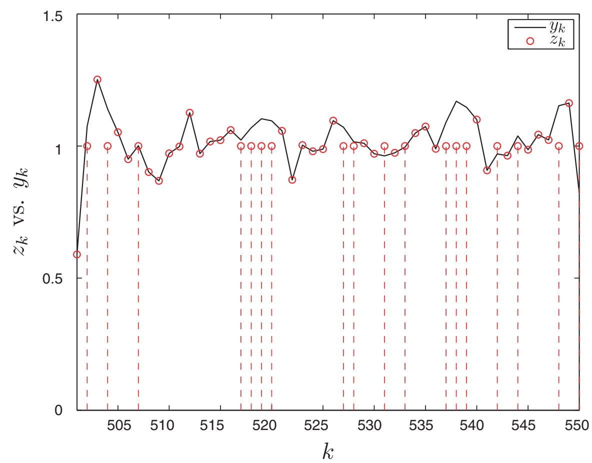

Figure 8. Example 2: comparison between estimated and true outputs when $\mu_k = 0.65$. (The dashed line represents output missing.)

that: at $k = 1500$, the output response has a big overshoot because of the parameter variation; however, the adaptive control scheme quickly forces the system output to track the desired signal again.

Observing Figures 4, 7, and 9 in three examples, we notice that the tracking error and oscillation still exist. This is mainly due to (1) the missing output measurements, and (2) the relatively high noise-signal ratio (around 25%). On the other hand, it is desirable to develop the new control schemes to further improve the control performance for networked systems subject to limited feedback information, which is worth to do extensive research.

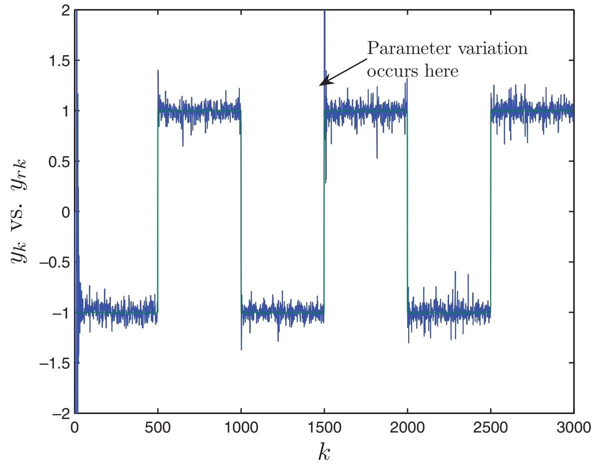

Figure 9. Example 3: output response subject to parameter variation: at time $k = 1500$, all parameters are increased by 50%.

6. CONCLUSION

This paper has investigated the problem of adaptive control for systems with SISO, ARX models placed in a network environment subject to unknown model parameters and randomly missing output measurements. The missing output estimator, Kalman filter-based model parameter estimator, and adaptive controller have been designed to achieve the output tracking. Convergence performance of the proposed algorithms is analyzed under the stochastic framework. Simulation examples verify the proposed methods. It is worth mentioning that the proposed scheme is developed for SISO systems in this work, and the extension to multi-input–multi-output systems is a subject worth further researching.

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